

Trigonometry

1. $\frac{2\sin x}{\cos x - \sin x \tan x} =$ A. $\tan 2x$ B. $\cot 2x$ C. $\tan x$ D. $\cot x$ E. $\sec x$
[2009F, A]
2. If $\sin \theta - \cos \theta = 0.2$ and $\sin 2\theta = 0.96$, find $\sin^3 \theta - \cos^3 \theta$. [2009S, 0.296]
3. In $\triangle ABC$, $AB = 5$, $BC = 9$, $AC = 7$. Find the value of $\frac{\tan \frac{A-B}{2}}{\tan \frac{A+B}{2}}$. [2008F, $\frac{1}{8}$]
4. In $\triangle ABC$, $AB = AC$ and in $\triangle DEF$, $DE = DF$. If AB is twice DE and $\angle D$ is twice $\angle A$, then the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$ is:
A. $\tan A$ B. $2\sec A$ C. $\csc 2A$ D. $\sec A \tan A$ E. $\cot 2A$ [2008F, B]
5. In hexagon $PQRSTU$, all interior angles $= 120^\circ$. If $PQ = RS = TU = 50$, and $QR = ST = UP = 100$, find the area of the triangle bounded by QT , RU , and PS to the nearest tenth. [2008F, 1082.5]
6. In $\triangle ABC$, $AB = AC = 25$, $BC = 14$. The perpendicular distances from a point P in the interior of $\triangle ABC$ to each of the three sides are equal. Find this distance. [2008S, $\frac{21}{4}$]
7. The graph of the function $f(x) = x + \sin kx$ ($|k| \leq 1$) intersects the graph of the function $f^{-1}(x)$ at $(4, a)$, $(12, b)$, and $(-8, c)$. Find the value of $a + b + c$.
[2007S, 8]
8. If $\cos(\arctan(x)) = x$ (x in radians), then x^2 can be expressed in the form $\frac{a + \sqrt{b}}{2}$.
Find $a + b$. [2007S, 4]
9. The sum of the solutions of $\arctan \frac{1}{x} + \arctan \frac{1}{x+2} = \arctan \frac{4}{x+4}$ is
A. negative B. even C. 1 D. greater than 5 E. prime [2007S, E]
10. Let $s(x) = \sin(\pi x)$ and $S(x) = [s(x)]^2$. Find $s(s(1/6)) + S(S(1/3))$. [2006F, 3/2]
11. In pentagon $AMTYC$, $AC = MT = 10$, $YT = CY = 20$, $\angle A = \angle M = 135^\circ$, and $\angle Y = 150^\circ$. Find the area of the pentagon to the nearest square unit. [2006F, 323]
12. If $f(x) = \cos \pi x$ and $g(x) = 2x$, find $f(g(1)) - g(f(1))$. [2006S, 3]
13. If $ABCD$, $DCEF$, $FEGH$ are squares with A, B, C, D, E, F, G and H all disjoint points, find $m\angle GAH + m\angle GDH + m\angle GFH$ to the nearest tenth of a degree.
[2006S, 90°]
14. In convex pentagon $AMTYC$, $\overline{CY} \perp \overline{YT}$, $\overline{MT} \perp \overline{YT}$, $CY = YT = 63$, $MT = 79$, $AM = 39$, and $AC = 52$. Find the area of the pentagon. [2006S, 5487]
15. If α is the acute angle formed by the lines with equations $y = 2x - 5$ and $y = 1 - 3x$, find $\tan \alpha$. [2006S, 1]
16. In the quadrilateral $PQRS$, $PQ = 1$, $QR = RS = \sqrt{2}$, $PS = \sqrt{3}$, and $QS = 2$. If T is the point of intersection of the diagonals, find the measure in degrees of angle RTS .
[2006S, 75]

17. $\triangle SML$ has sides of length 6, 7, 8. Find the exact value of $(\cos S + \cos M + \cos L)$.
[2005F, $\frac{47}{32}$]
18. Find the sum of all solutions of $\cos x = \cot x \cos x$ for which $0 \leq x \leq 2\pi$. [2005F, 3.5π]
19. A triangle has vertices $A(0,0)$, $B(3,0)$, and $C(3,4)$. If the triangle is rotated counterclockwise around the origin until C lies on the positive y -axis, find the area of the intersection of the region bounded by the original triangle and the region bounded by the rotated triangle. [2005F, $\frac{21}{16}$]
20. If $0 < t < \pi/2$, $0 < z < 1$, and $\cos t = \frac{1-z^2}{1+z^2}$, how many of the following are true?
 $z = \sqrt{\frac{1-\cos t}{1+\cos t}}$; $\sin t = \frac{2z}{1+z^2}$; $\tan t = \frac{2z}{1-z^2}$; $z = \tan \frac{t}{2}$ [2005F, 4]