

### Combinatorics/Probability (Part I)

1. A fair coin is labeled A on one side and M on the other; a fair die has two sides labeled T, two labeled Y, and two labeled C. The coin and die are each tossed three times. Find the probability that the six letters can be arranged to spell AMATYC. [2008S,  $\frac{1}{12}$ ]
2. The letters AMATYC are written in order, one letter to a square of graph paper, to fill 100 squares. If three squares are chosen at random without replacement, find the probability to the nearest 1/10 of a percent of getting three A's. [2008S, 0.037007]
3. A student committee must consist of two seniors and three juniors. Five seniors are able to serve on the committee. What is the least number of junior volunteers needed if the selectors want at least 600 different possible ways to pick the committee? [2008S, 9]
4. How many different 3-letter strings can be formed from the letters of MATHEMATICS (no letter can be used in a given string more times than it appears in the word)? [2007S, 399]
5. A positive integer less than 1000 is chosen at random. What is the probability it is a multiple of 3, but a multiple of neither 2 nor 9. [2006S,  $\frac{1}{9}$ ]
6. A palindrome is a word or a number (like RADAR or 1221) which reads the same forwards and backwards. If dates are written in the format MMDDYY, how many dates in the 21<sup>st</sup> century are palindromes? [2005S, 24]
7. Mrs. Abbott finds that the number of possible groups of 3 students in her class is exactly five times the number of possible groups of 2 students. How many students are in her class? [2005S, 17]
8. Teams A and B play a series of games; whoever wins two games first wins the series. If Team A has a 70% chance of winning any single game, what is the probability that Team A wins the series? [2004S, 0.784]
9. A piece has 2 saxophone parts, 3 trumpet parts, and 3 trombone parts. If a band has 2 saxophonists, 3 trumpeters, and 3 trombonists, in how many ways can different parts be assigned to each player? [2007F, 72]
10. Five students enroll in a statistics class. The first test is scored on a percent basis (0% to 100%) rounding each score to the nearest whole number. Four of their scores are 93, 96, 99, and 100. How many possible whole number scores on the fifth student's test will make the median of the five scores equal to the mean of the five scores?  
A. 0   B. 1   C. 2   D. 3   E. more than 3   [2006F, C]
11. A basketball player has a constant probability of 80% of making a free throw. Find the probability that her next successfully free throw is the third or fourth one she attempts. [2006F, 0.0384]
12. If you have eight pairs of socks, each pair a different color, find the probability that if you randomly lose five socks, the remaining socks form exactly four matching pairs (and three unmatched socks). [2006F,  $\frac{20}{39}$ ]
13. A circle contains 25 points chosen so that the arcs between any two adjacent points are equal. Three of these points are chosen at random. Let the probability that the triangle formed is right be  $R$ , and the probability that the triangle formed is isosceles be  $I$ . Find  $|R - I|$ . [2005F,  $\frac{3}{23}$ ]

14. Let  $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ . How many three-element subsets of  $A$  contain at least two consecutive integers? [2004F, 64]
15. A store has four open checkout stands. In how many ways could six customers line up at the checkout stands? [2004F, 60480]
16. Consider all arrangements of the letters AMATYC with either the A's together or the A's on the ends. What fraction of all possible such arrangements satisfies these conditions? [2003F,  $\frac{2}{5}$ ]