## Algebra (Part I)

1. For how many different integers b is the polynomial $x^2 + bx + 16$ factorable over the integers? [2008S, 6]

2. Let $f(x) = x^2 - 2x + 4$. Which of the following is a factor of $f(x) - f(2y)$?
   (A) $x + 2y$ (B) $x + 2y + 2$ (C) $x - 2y + 2$ (D) $x + 2y - 2$ (E). none of these.
   [2008S, D]

3. What is the value of $(\log_{624} 625)(\log_{623} 624)(\log_{622} 623)\ldots(\log_6 7)(\log_5 6)$?
   [2008S, 4]

4. Ed drives to work at a constant speed S. One day he is halfway to work when he immediately turns around, speeds up by 8 mph, and drives home. As soon as he is home, he turns around and drives to work at 6 mph faster than he drove home. His total driving time is exactly 67% greater than usual. Find S in mph. [2008S, 42]

5. One can of frozen juice concentrate, when mixed with 4 1/3 cans of water, makes 2 quarts (64 oz) of juice. Assuming no volume is grained or lost by mixing, how many oz does a can hold. [2007F, 12]

6. Define the operation Δ by $a\Delta b = ab + b$. Find $(3\Delta 2)\Delta(2\Delta 3)$. [2007F, 81]

7. A square is covered by a design made up of four identical rectangles surrounding a central square, as shown at the right. If the area of the central square is 4/9 of the area of the entire design, find the ratio of the length of a rectangle to the side of the central square. [2007F, $\frac{5}{4}$]
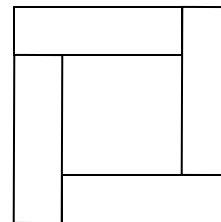
8. Square $PQRS$ has sides of length 10. Points T, U, V, W are chosen on sides PQ, QR, RS, and SP respectively so that $PT = QU = RV = SW = 2$. Find the area of quadrilateral $TUVW$. [2007F, 68]

9. A bicycle travels at s feet/min. When its speed is expressed in inches/sec, the numerical value decreases by 16. Find s. (1 foot = 12 inches) [2007F, 20]

10. A door is 4 ft wide and 7 ft high. If the door is standing open at a $90°$ angle with the door frame, what is the greatest distance in feet from the outer top corner of the door to a point on the door frame? [2007F, 9]

11. A class is exactly 40% female. When 3 male students are replaced by female students, the class becomes exactly 44% female. How many more males than females are in the original class? [2007F, 15]

12. The average of any 17 consecutive perfect square integers is always k greater than a perfect square. If $k = 2^r m$, where m is odd, find r. [2007F, 3]

13. If $f(x) = 3x - 1$ and $g(x) = x^2$, find $g(f(f(1)))$. [2007S, 25]

14. The teachers at Oak Tech have cars with average mileage 39000 miles. George buys a brand-new car, keeping his old car, and the average mileage drops to 36400. How many cars do the teachers now own? [2007S, 15]

15. The sequence $\log x$, $\log x^2$, $\log x^3$, $\log x^4$, … is best described as which of the following? A. geometric with common ratio $\log x$. B. geometric with common ratio x C. arithmetic with common difference $\log x$. C. arithmetic with common difference x. E. neither geometric nor arithmetic. [2007S, C]

16. If $\ln s = 0.6$ and $\ln t = 0.9$, find $\log_{st} e^{5.4}$. [2007S, 3.6]

17. A function is symmetric to the origin and periodic with period 8. If $f(2)=3$, what is the value of $f(4)+f(6)$? [2007S, -3]

18. For how many integer values of k do the graphs of $x+y=k$ and $xy=k$ NOT intersect? [2007S, 3]

19. A point is chosen at random from the interior of a square of side 16. Find the probability that the point is at least $\sqrt{2}$ units from both diagonals. [2007S, $\frac{9}{16}$]

20. A jug holds 10 gal of antifreeze. I fill an empty bottle from the jug and replace the amount I poured out with water, mixing well. I refill the emptied bottle again from the jug, refilling the jug with water and mixing well, and then repeat this process once more. The bottle is now half water. To the nearest tenth of a gallon, what is the volume of the bottle? [2007S, 2.9]

21. A farmer plants A acres of wheat one year. Each year thereafter, he harvests (removes) $1/4$ of the planted acreage and then plants 1500 more acres. The number of acres of wheat planted approaches what number? [2007S, 6000]

22. If $f(x)=\dfrac{x^2-3x-4}{x+1}$, the inverse of $f(x)$ can be written as $f^{-1}(x)=\dfrac{x^2+ax+b}{x+c}$.
Find $a+b+c$. [2007S, 34]

23. Choose k so that the system $\begin{cases} x+y+kz=1 \\ x+ky+z=2 \\ kx+y+z=-3 \end{cases}$ is dependent. For which pair $(x,\ y)$ below does there exist a z such that $(x,\ y,\ z)$ will satisfy the resulting dependent system?
    A. $(\frac{7}{3},0)$ B. $(3,\frac{2}{3})$ C. $(\frac{8}{3},1)$ D. $(\frac{4}{3},-1)$ E. $(\frac{1}{3},-2)$ [2007S, C]

24. The numerical values of the x- and y-intercepts of a straight line are the same nonzero number. What is the slope of the line? [2006F, -1]

25. Sue works weekdays for $10 an hour, Saturdays for $15 an hour, and Sundays for $20 an hour. If she worked 180 hours last month and earned $2315, how many more weekday hours than Sunday hours did she work last month? [2006F, 77]

26. The equation $x^{\log_{25}9}+9^{\log_{25}x}=54$ has a solution in common with which of the following?
    A. $x^3-125x^2-x+125=0$ B. $x^3+5x^2-25x-125=0$
    C. $x^3-5x^2-25x+125=0$ D. $5x^3+5x^2-125x-125=0$
    E. $5x^3-5x^2-125x+125=0$ [2006F, A]

27. If $h(x)=2x+2$ and $k(x)=2x^3-7x^2-11x+6$, find the sum of all of the irrational zeros of $h(k(x))$ and $k(h(x))$. [2006F, 7/2]

28. If $h(x)=2x+2$ and $k(x)=2x^3-7x^2-11x+6$, find the sum of all of the rational zeros of $h(k(x))$ and $k(h(x))$. [2006F, -5/4]